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GENERATOR COORDINATE CALCULATIONS OF TWO-NUCLEON MOMENTUM DISTRIBUTIONS IN ^{4}He, ^{16}O AND ^{40}Ca NUCLEI

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The two-nucleon centre-of-mass and relative motion momentum distributions for n-p pairs inside the ^{4}He, ^{16}O and ^{40}Ca nuclei are derived from the two-body density matrix obtained in an approach within the generator coordinate method. Square-well as well as harmonic oscillator construction potentials and Skyrme-like forces are used in the calculations. The calculated momentum distributions are compared with the results from other correlation approaches. The presence of high-momentum components in the two-nucleon momentum distributions in the particular case of square-well construction potential is due to the effective account of short-range nucleon-nucleon correlations in the approach.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Расчет двухнуклонных импульсных распределений в ядрах ^{4}He, ^{16}O и ^{40}Ca методом генераторной координаты

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На основе двухчастичной матрицы плотности методом генераторной координаты рассчитаны импульсные распределения центра масс и относительного движения n–p пары в ядрах ^{4}He, ^{16}O и ^{40}Ca. В подходе использованы эффективные силы Скирма и конструкционные потенциалы гармонического осциллятора и бесконечно глубокой прямоугольной потенциальной ямы. Полученные импульсные распределения сравниваются с результатами других корреляционных подходов. Наличие высокоимпульсных компонент в двухчастичных распределениях в случае бесконечно глубокого прямоугольного конструкционного потенциала связано с эффетивным учетом короткодействующих нуклон-нуклонных корреляций в подходе.

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1. Introduction

There is an increasing interest in the nucleon momentum distributions (NMD) in nuclei, especially in the high-momentum region, where this nuclear characteristic reflects the presence of short-range nucleon-nucleon correlations (SRC) in nuclei. The high-momentum component of the NMD is very sensitive to the SRC and can be investigated in different nuclear processes, such as deep-inelastic scattering of leptons, protons, pions, alpha-particles and heavy ions in nuclei, nuclear photo-effect, etc. /1/ . However, there is scanty knowledge about the behaviour of the NMD at high momenta nowadays. The extracted information from the experiments is dependent on the suggested model and reaction mechanism. The main difficulties in the theory are connected with the consistent account of the SRC in the nuclear models /2-5/ . It can be noted that the application of the most promising theoretical correlation schemes to nuclei heavier than some light nuclei is very complicated. An approach which accounts for SCR and can be applied to both light and heavy nuclei without significant difficulties is proposed in the coherent density fluctuation model (CDFM) /1,6/ . The basic relations of the CDFM are obtained by a generalization of the delta-function limit in the generator coordinate method (GCM) /7/. The presence of high-momentum components in the single-particle NMD evaluated in the CDFM is due to the particular choice of the intermediate generating states, allowing a certain type of SRC in the nuclei to be accounted for.

Recently the CDFM was extended for evaluation of the two-nucleon centre-of-mass and relative motion momentum distributions /8/. The extended model was applied to the ^{4}He, ^{16}O and ^{40}Ca nuclei. Similar to the case of the single-nucleon momentum distribution, the two-nucleon momentum distributions (TNMD) calculated within the CDFM have high-momentum components which should be important for the description of various nuclear reactions, such as two-nucleon mechanism of π^- -absorption, backward proton production in proton-nucleus collisions, etc. The dominant role which the two-nucleon centre of mass momentum distribution plays for the successful description of the high-energy part of the back-scattered proton spectra in proton-nucleus collision is shown by Haneishi and Fujita /9/. They use momentum distributions with high-momentum tails in their calculations. The prominent high-momentum components of the TNMD in ^{4}He nucleus coming from the account of correlations are derived also by Akaishi /10/ in calculations with Reid soft core potential using the ATMS method (an abbreviation of Amalgamation of Two-body correlations into Multiple Scattering process).

The role of the delta-function limit used in the CDFM on the nucleon momentum distribution has been studied [11,12] in the framework of the generator coordinate method [7]. Skyrme-like effective forces and different construction potentials have been used in the calculations of the NMD in ^{4}He, ^{16}O and ^{40}Ca.

The purpose of the present work is to calculate the two-nucleon momentum distributions in the GCM approach, avoiding the delta-function approximation in the CDFM with respect to the TNMD. We use the two-body density matrix obtained in the GCM with square-well and harmonic oscillator construction potentials as well as Skyrme-like forces in this approach. In Sect.2 a brief review of the main relations in the GCM are given. The derivation of the two-body density matrix and TNMD for n-p pairs inside the nucleus in the framework of the GCM are presented in Sect.3. In Sect.4 the calculations of the TNMD in ^{4}He, ^{16}O and ^{40}Ca nuclei are presented in both the cases of harmonic oscillator and square-well construction potentials. The comparison with other theoretical results is also presented and discussed.

2. GCM General Relations

In the GCM the wave function Ψ for an A-particle system is written in the form [7]:

$$\Psi(\vec{r}_1,\vec{r}_2,\dots,\vec{r}_A) = \int d\alpha\, f(\alpha)\,\Phi(\alpha;\vec{r}_1,\vec{r}_2,\dots,\vec{r}_A)\ , \qquad (1)$$

where $\Phi(\alpha;\{\vec{r}_i\})$ $(i = 1, 2, \dots, A)$ is the generating function and the unknown weight function $f(\alpha)$ is determined by solving the Griffin-Hill-Wheeler equation:

$$\int [\mathcal{H}(\alpha,\alpha') - E\, I(\alpha,\alpha')]\, f(\alpha')\, d\alpha' = 0\ . \qquad (2)$$

The overlap integral I and the energy kernel $\mathcal{H}$ in eq.(2) are of the forms

$$I(\alpha,\alpha') = \langle\Phi(\alpha;\{\vec{r}_i\})\mid\Phi(\alpha';\{\vec{r}_i\})\rangle\ , \qquad (3)$$

$$\mathcal{H}(\alpha,\alpha') = \langle\Phi(\alpha;\{\vec{r}_i\})|\hat{H}|\Phi(\alpha';\{\vec{r}_i\})\rangle, \qquad (4)$$

and $\hat{H}$ is the hamiltonian of the system.

Following the GCM approach proposed in [11,12], the generating wave function $\Phi(\alpha;\{\vec{r}_i\})$ is taken to be a Slater determinant constructed from neutron and proton orbitals $\phi_\lambda(\alpha,\vec{r})$ $(\lambda = 1, 2, \dots, A/4)$, which correspond to a given construction potential. We consider two different potentials in this work — a square-well with infinite walls and a harmonic oscillator. In these cases the generator coordinate is the radius of the well and the oscillator parameter, correspondingly. When Skyrme-like forces are used, the energy kernel $\mathcal{H}$ in eq.(4) takes the form /13/:

$$\mathcal{H}(\alpha, \alpha') = I(\alpha, \alpha') \int H(\alpha, \alpha', \vec{r})\, d\vec{r}. \tag{5}$$

Further we shall restrict our considerations to spin saturated N = 2 nuclei and neglecting the spin-orbit and Coulomb forces. In this case $H(\alpha, \alpha', \vec{r})$ is given by /14/:

$$H(\alpha, \alpha', \vec{r}) = \frac{h^2}{2m} T + \frac{3}{8} t_0 \rho^2 + \frac{1}{16}(3t_1 + 5t_2)(\rho T + \vec{j}^{\,2}) + \\ + \frac{1}{64}(9t_1 - 5t_2)(\nabla\rho)^2 + \frac{1}{16} t_3 \rho^{2+\sigma}, \tag{6}$$

where the quantities t_0, t_1, t_2, t_3 and σ are the Skyrme-force parameters. The density ρ, the kinetic energy density T and the current density $\vec{j}$ are defined by

$$\rho(\alpha, \alpha', \vec{r}) = 4 \sum_{\lambda,\mu=1}^{A/4} (N^{-1})_{\mu\lambda}\, \phi^*_\lambda(\alpha, \vec{r})\, \phi_\mu(\alpha', \vec{r}), \tag{7}$$

$$T(\alpha, \alpha', \vec{r}) = 4 \sum_{\lambda,\mu=1}^{A/4} (N^{-1})_{\mu\lambda}\, \nabla\phi^*_\lambda(\alpha, \vec{r}) \cdot \nabla\phi_\mu(\alpha', \vec{r}), \tag{8}$$

$$\vec{j}(\alpha, \alpha', \vec{r}) = 2 \sum_{\lambda,\mu=1}^{A/4} (N^{-1})_{\mu\lambda} \{\phi^*_\lambda(\alpha, \vec{r})\, \nabla\phi_\mu(\alpha', \vec{r}) - (\nabla\phi^*_\lambda(\alpha, \vec{r}))\,\phi_\mu(\alpha', \vec{r})\}, \tag{9}$$

where

$$N_{\lambda\mu} = \int d\vec{r}\, \phi^*_\lambda(\alpha, \vec{r})\, \phi_\mu(\alpha', \vec{r}), \tag{10}$$

and the overlap kernel (3) is given by

$$I(\alpha, \alpha') = [\det(N_{\lambda\mu})]^4. \tag{11}$$

In order to solve the Griffin-Hill-Wheeler equation (2) we use a discretization procedure similar to that of ref. /14/. Thus eq.(2) is reduced to a matrix eigenvalue problem:

$$\sum_i [\mathcal{H}(a_j, a_i') - E\, I(a_j, a_i')]\, f(a_i') = 0 . \tag{12}$$

The c.m. correlations are taken into account as in /14/.

3. Two-Nucleon Momentum Distributions

We consider the GCM wave function Ψ for an A-particle system written in the form:

$$\Psi(\xi_1, \xi_2, \dots, \xi_A) = \int da\, f(a)\, \Phi(a; \xi_1, \xi_2, \dots, \xi_A), \tag{13}$$

where each coordinate ξ_i (i = 1, 2, ..., A) stands for the space coordinate r_i, the spin coordinate s_i and the coordinate of the isotopic spin τ_i, i.e. $\xi_i \equiv (\vec{r}_i; \sigma_i, \tau_i) = (\vec{r}_i; \eta_i)$ with $\eta_i \equiv (\sigma_i, \tau_i)$; a is the generator coordinate, and the function Φ is the generating function.

In order to derive an expression for the TNMD in GCM we start with the two-body density matrix:

$$\rho^{(2)}(\xi_1, \xi_2; \xi_1', \xi_2') = \\ = \frac{1}{2} A(A-1) \sum_{\eta_3 \dots \eta_A} \int d\vec{r}_3 \dots d\vec{r}_A \Psi^+(\xi_1, \xi_2, \xi_3 \dots \xi_A)\, \Psi(\xi_1', \xi_2', \xi_3 \dots \xi_A). \tag{14}$$

Using the many-particle wave function (13), the two-body density matrix in GCM can be expressed in the form

$$\rho^{(2)}(\xi_1, \xi_2; \xi_1', \xi_2') = \\ = \int da\, f^*(a) \int da'\, f(a')\, \rho^{(2)}(a, a'; \xi_1, \xi_2; \xi_1', \xi_2'), \tag{15}$$

where

$$\rho^{(2)}(a, a'; \xi_1, \xi_2; \xi_1', \xi_2') = \\ = \frac{1}{2} A(A-1) \sum_{\eta_3 \dots \eta_A} \int d\vec{r}_3 \dots d\vec{r}_A \Phi^+(a; \xi_1, \xi_2, \xi_3 \dots \xi_A)\Phi(a'; \xi_1', \xi_2', \xi_3 \dots \xi_A). \tag{16}$$

If the generating wave functions Φ are Slater determinants constructed from an orthonormal and complete set of one-particle functions $\phi_i(a, \xi)$, the matrix (16) can be expressed by the quantity $\rho(a, a'; \xi, \xi')$ /15/ :

$$\rho^{(2)}(a,a'; \xi_1, \xi_2; \xi'_1, \xi'_2) = \frac{I(a,a')}{2!} \begin{vmatrix} \rho(a,a'; \xi_1, \xi'_1) & \rho(a,a'; \xi_1, \xi'_2) \\ \rho(a,a'; \xi_2, \xi'_1) & \rho(a,a'; \xi_2, \xi'_2) \end{vmatrix} \tag{17}$$

where

$$\rho(a,a'; \xi, \xi') = \sum_{k,\ell=1}^{\hat{}} \phi^*_k(a, \xi)\, \phi_\ell(a', \xi')\, (N^{-1})_{\ell k} \, . \tag{18}$$

In (18) $(N^{-1})_{\ell k}$ is the inverse matrix to the matrix $N_{k\ell}$ defined by

$$N_{k\ell} = \sum_\eta \int d\vec{r}\, \phi^*_k(a, \xi)\, \phi_\ell(a', \xi) \tag{19}$$

and $I(a, a')$ is the overlap integral from eq.(3):

$$I(a, a') = \det(N_{k\ell}) \, . \tag{20}$$

Thus we get the following expression for the two-body density matrix in the GCM:

$$\rho^{(2)}(\xi_1, \xi_2; \xi'_1, \xi'_2) = \frac{1}{2} \int da\, f^*(a) \int da'\, f(a')\, I(a, a') \times \\ \times [\rho(a, a'; \xi_1, \xi'_1)\, \rho(a,a'; \xi_2, \xi'_2) - \rho(a, a'; \xi_1, \xi'_2)\, \rho(a,a'; \xi_2, \xi'_1)]. \tag{21}$$

The two-nucleon momentum distribution $n^{(2)}(\zeta_1, \zeta_2)$ is related to the diagonal elements of the two-body density matrix in the momentum space

$$n^{(2)}(\zeta_1, \zeta_2) = \tilde{\rho}^{(2)}(\zeta_1, \zeta_2; \zeta_1, \zeta_2), \tag{22}$$

where $\zeta_i \equiv (\vec{k}_i, \sigma_i, \tau_i) = (\vec{k}_i, \eta_i)$ with $\eta_i \equiv (\sigma_i, \tau_i)$, and $\vec{k}_i$ is the momentum of the i-th nucleon.

Further we restrict our considerations to spin saturated N = Z nuclei. After summation over $\sigma_1, \sigma_2, \sigma_1$ and τ_2 $(\tau_2 \neq \tau_1)$ we get for the case of n-p pairs:

$$n^{(2)}_{np}(\vec{k}, \vec{k}) = \frac{1}{4} \int d\alpha\, f^*(\alpha) \int d\alpha' f(\alpha') I(\alpha, \alpha')\, \tilde{\rho}(\alpha, \alpha; \vec{k}_1)\, \tilde{\rho}(\alpha, \alpha'; \vec{k}_2), \tag{23}$$

where

$$\tilde{\rho}(\alpha, \alpha'; \vec{k}) \equiv 4 \sum_{\lambda, \mu=1}^{A/4} (N^{-1})_{\mu\lambda}\, \tilde{\phi}^*_\lambda(\alpha, \vec{k})\, \tilde{\phi}_\mu(\alpha', \vec{k}), \tag{24}$$

$\tilde{\phi}_\lambda(\alpha, \vec{k})$ are the Fourier transform of the orbitals $\phi_\lambda(\alpha, \vec{r})$, and each orbital state is occupied by 4 nucleons.

Introducing the relative $\vec{q}$ and the center of mass $\vec{p}$ momenta of the n-p pair by

$$\vec{q} = (\vec{k}_1 - \vec{k}_2)/2; \quad \vec{p} = \vec{k}_1 + \vec{k}_2, \tag{25}$$

the following expressions for the center of mass and the relative motion TNMD for n-p pairs inside the nucleus can be obtained

$$n^{c.m.}_{np}(\vec{p}) = \int \frac{d\vec{q}}{(2\pi)^3} n^{(2)}_{np}\left(\frac{\vec{p}}{2} + \vec{q}, \frac{\vec{p}}{2} - \vec{q}\right), \tag{26}$$

$$n^{rel.}_{np}(q) = \int \frac{d\vec{p}}{(2\pi)^3} n^{(2)}_{np}\left(\frac{\vec{p}}{2} + \vec{q}, \frac{\vec{p}}{2} - \vec{q}\right). \tag{27}$$

Both distributions (eqs. (26) and (27)) are normalized to $A^2/4$.

4. Results and Discussion

The TNMD $n^{c.m.}_{np}$ (eq. (26)) and n^{rel}_{np} (eq.(27)) are calculated in the cases of ^{4}He, ^{16}O and ^{40}Ca nuclei within the proposed GCM-scheme and using both square-well and harmonic oscillator construction potentials. The values of the Skyrme parameters in (6) for the case of square-well construction potential were taken from ref. /12/, where they are determined to fit the binding energies of ^{4}He, ^{16}O and ^{40}Ca (t_0 = -2765.0, t_1 = 383.94, t_2= -38.04, t_3 = 15865 and σ= 1/6). In the case of harmonic oscillator construction potential, the SkM* parameter set is used /16/.

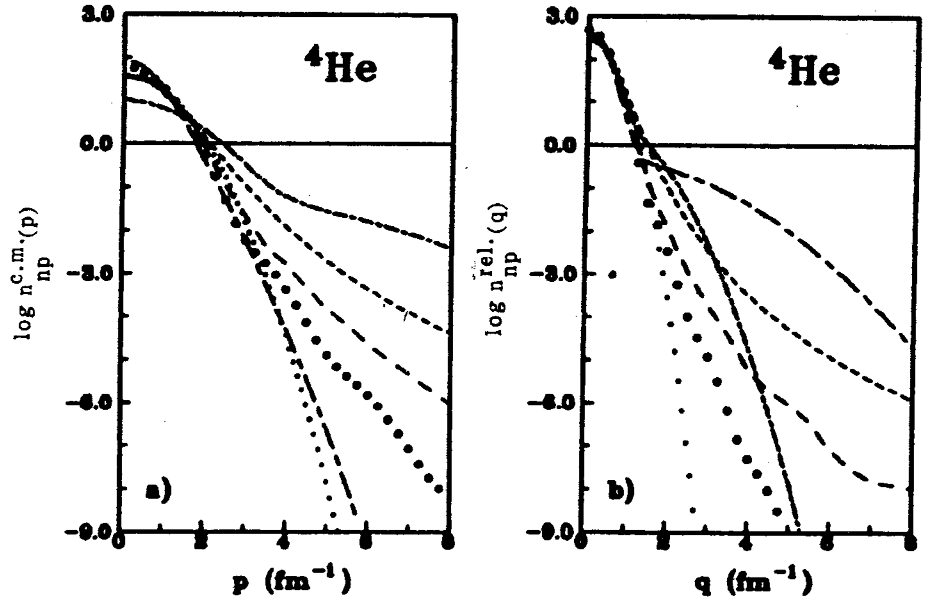

Fig.1. Two-nucleon centre-of-mass (a) and relative state (b) momentum distributions of n-p pairs in ^{4}He. ······· harmonic-oscillator shell-model result; • • • • • GCM with harmonic-oscillator construction potential; — — — — GCM with square-well construction potential; ------ the CDFM result /8/ ; —•—•—•— phenomenological distributions from /9/; -•-•-•- the ATMS result from /10/. The normalization is $\int n_{np}(\vec{k})\, d\vec{k}/(2\pi)^3 = 1$.

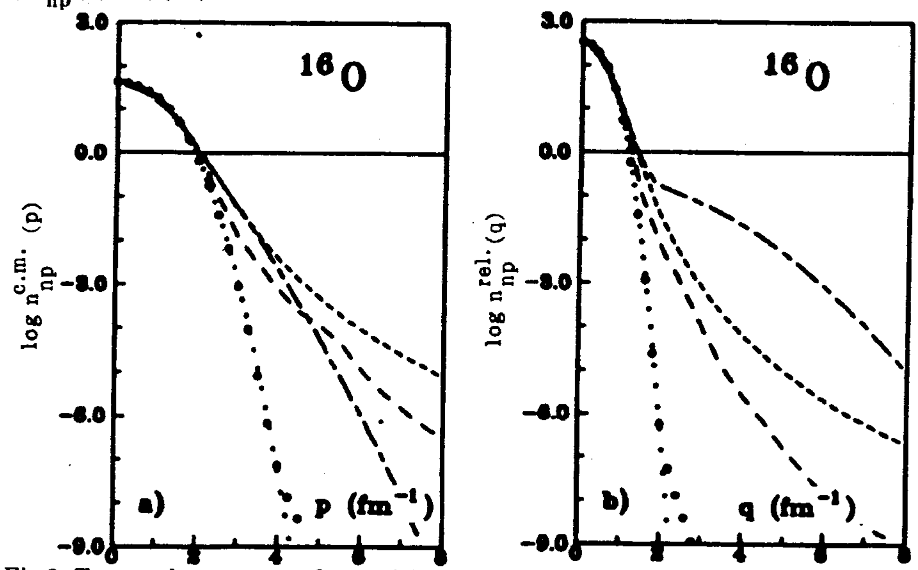

Fig.2. Two-nucleon centre-of-mass (a) and relative state (b) momentum distributions of n-p pairs in ^{16}O. ······· harmonic-oscillator shell-model result; • • • • • GCM with harmonic-oscillator construction potential; — — — — GCM with square-well construction potential; ------ the CDFM result /8/ ; —•—•—•— phenomenological distributions from /9/. The normalization is $\int n_{np}(\vec{k})\, d\vec{k}/(2\pi)^3 = 1$.

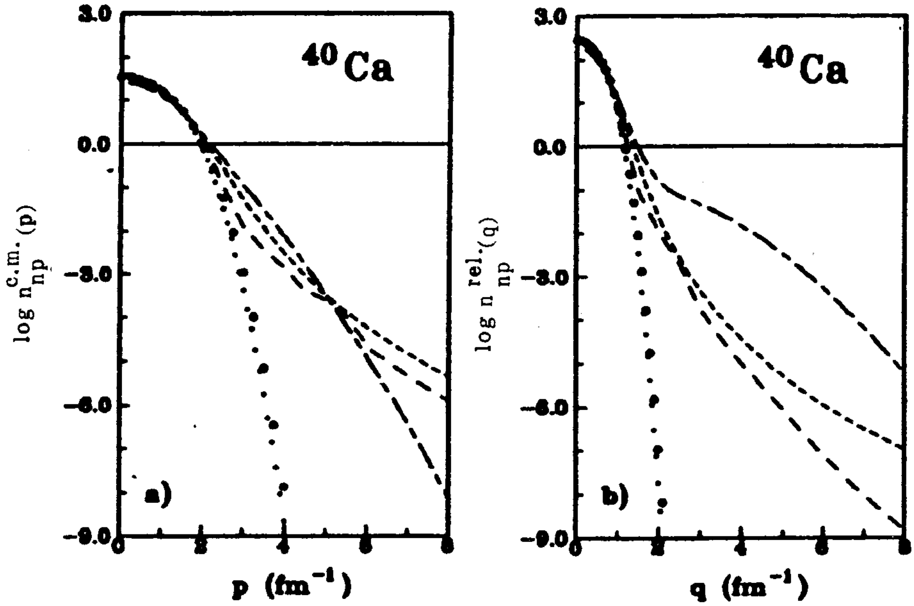

Fig.3. Two-nucleon centre-of-mass (a) and relative state (b) momentum distributions of n-p pairs in ^{40}Ca. ······· harmonic-oscillator shell-model results; • • • • • • GCM with harmonic-oscillator construction potential; — — — — — GCM with square-well construction potential; ------- the CDFM result /8/; —•—•—•— phenomenological distributions from /9/. The normalization is $\int n_{np}(\vec{k})\, d\vec{k}/(2\pi)^3 = 1$.

The Griffin-Hill-Wheeler equation (2) is reduced to a matrix eigenvalue problem (eq. (12)) and the lowest solution $f_0(\alpha)$ is substituted in eq. (23) in order to calculate the TNMD's ((26) and (27)) in the nuclear ground state.

The calculated TNMD's of ^{4}He, ^{16}O and ^{40}Ca are presented in Figs. 1, 2 and 3, respectively. The calculated momentum distributions are compared with those obtained in the model from ref. /9/, as well as with those from CDFM calculations /8/. In the case of ^{4}He nucleus the comparison with variational ATMS calculations /10/ with Reid soft-core N-N interaction is made.

We note that the existence of high-momentum components in the two-nucleon momentum distributions in the particular case of square-well construction potential is an evidence for the effective account of short-range nucleon-nucleon correlations in this approach within the generator coordinate method. This fact is obviously due to the presence of intermediate states in the GCM with a high density (at small values of the generator coordinate x) at which the nucleons are close to each other and the SRC are operative.

In conclusion it has to be emphasized that the experimental investigations concerning the two-nucleon momentum distributions would be of great interest.

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